Articles

On the Dynamics of Mid-Latitude Surface Westerlies

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1. Introduction

The atmosphere is a thin layer of gas surrounding the Earth. Its motions are complex and chaotic – it is not possible to accurately predict the weather more than a few days ahead. Nevertheless, when large space and time scales are considered there are certain coherent features that can be observed. Together these features constitute what we call the general circulation. The modern view regards the general circulation as consisting of three 'cells' based on the mean meridional mass transport in the atmosphere. Figure 1 shows the main features of this cell picture. In the tropics there exist the Hadley cells. Tropical air, warmed by latent heat release, rises near the equator, radiatively cools, and descends in the subtropics. Since warmer air is rising, and cooler air sinking, the Hadley cell is a thermally direct circulation. A weak thermally direct cell also exists at polar latitudes. In the extra-tropics however, the mean meridional overturning is in the opposite direction. Named the Ferrel cell, this circulation accomplishes much of its transport of momentum and heat via large scale eddies.
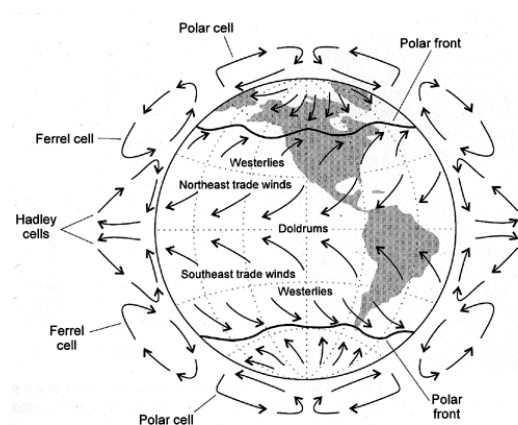

Figure 1: A schematic of the main features of the general circulation. The atmosphere is broken into three latitude belts in each hemisphere, corresponding to the three `cells' of meridional overturning. The mid-latitude cell – the Ferrel cell – is thermally indirect, and has prevailing surface westerlies beneath it. Source: Datta (2008).

Closely coupled to the meridional circulation is that of the pattern of winds. The most striking feature of the mean zonal winds is the subtropical jets in the upper atmosphere (see figure 3). These regions of strong westerlies (from the west) are directly driven by the pole-to-equator temperature gradient. However, this essay is concerned with the prevailing surface winds. While much weaker than the upper level jets, the surface winds do exhibit some latitudinal structure. In the tropics and polar latitudes the low-level winds are easterly, while in mid-latitudes they are westerly. This essay examines the dynamical processes that lead to the maintenance of these surface westerly winds in the atmosphere. We will see that the surface winds are intimately linked to large-scale eddy motions and the upper level flow. Thus, in order to explain the mid-latitude westerlies we will need to describe the extra-tropical circulation as a whole.

Theories of the general circulation trace back to the single celled picture of George Hadley. Since then, our understanding has progressed with a series of revolutions, each being based on new observations that contradict the old view (Lorenz, 1983). Hadley's single cell was replaced by the two-cell model of William Ferrel, which subsequently paved the way for the modern view in the early parts of the 20th century. In section 2 we give an account of these developments in order to give some context to the discussion in the rest of the essay. Progress on the theoretical side, however, would be almost impossible if it were not for the immense increase in our observational abilities over the last 100 years. We discuss the general circulation from the observational viewpoint in section 3. In section 4 we analyse, through simple conceptual models, how large scale eddies act to maintain the extra-tropical circulation, and produce the surface westerlies. We conclude by discussing current developments in the field of large-scale atmospheric dynamics and some of the open problems still left to be solved.

2. History of theories of the general circulation

Attempts to explain what we now describe as the atmospheric general circulation go back more than three centuries. There have been

numerous theories proposed based on the data available at the time. Almost invariably, these are subsequently proved to be false by new observations (Lorenz, 1983). In this section we focus on a small subset of these theories that have had a lasting impact on the field, and form part of the story of how our understanding of the general circulation has evolved. We will then be in a position to examine in more detail the extra-tropical circulation and address the title question; why are the prevailing surface currents at middle latitudes toward the east?

2.1. Single celled models

The first exposition on the general circulation of the atmosphere that explicitly offered an explanation of the westerly currents at mid-latitudes was in 1735, in a now famous paper by George Hadley. Hadley argued that warm air at the equator would rise due to its low density, flowing out in the upper atmosphere, and sinking at the poles. As air parcels moved around this meridional overturning circulation they would conserve their absolute velocity (Lorenz, 1967). Since the rotational velocity of the Earth is largest at the equator, and drops to zero at the poles, air parcels drawn equatorward by the meridional circulation would be deflected westwards relative to the Earth's surface. If this velocity conservation is exact, calculation shows that the zonal velocities in the subtropics will be much greater than observed. Hadley explains the difference as due to friction with the Earth's surface. Quantitatively this argument is not accurate – air parcels that experience no zonal forces conserve their angular momentum and not their velocity in motion around the Earth (Schneider, 2006). Nevertheless the qualitative conclusions are the same as those if we assume angular momentum conservation.
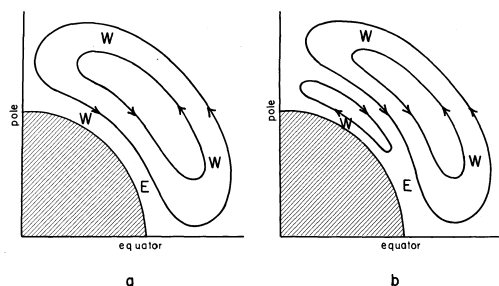

Figure 2: Models of the meridional atmospheric circulation due to (a) Hadley and (b) Thomson. Streamlines show the direction of the meridional circulation, while letters E and W give the direction – easterly or westerly – of the prevailing zonal winds. Source: Lorenz (1967).

The surface westerlies at higher latitudes are treated similarly in Hadley's model. Air parcels in the poleward moving branch of the circulation are moving down the gradient of planetary velocity, and hence must acquire a westerly relative velocity. Thus, at the sinking regions there will be strong westerly surface winds which will decrease in magnitude, and then become easterly, as one moves back toward the equator. Hadley correctly notes that the friction on the westerlies must balance the friction on the easterlies otherwise the Earth's rotation rate would be changing. A schematic of the circulation he envisaged is shown in figure 2 (a).

As with most attempts to explain the general circulation of the atmosphere prior to the modern era, Hadley's description was limited by the observations of the day. Just at a time when his model was gaining some acceptance, new observations showed its predictions to be incorrect. The westerly winds in the extra-tropics have a poleward component rather than an equatorial component (Lorenz, 1983). Attempts to explain this drift led to the beginnings of multi-celled models of the atmospheric general circulation such as that made by Ferrel.

2.2. Toward a multi-celled general circulation

A possible reconciliation between Hadley's theory and the conflicting wind observations came about due to considerations of the meridional component of the Coriolis force made independently by William Ferrel and James Thomson (Ferrel, 1859; Thomson, 1857). Both authors invoked a shallow thermally indirect cell at higher latitudes so that the surface currents would match the observations. The larger, thermally direct cell was allowed to continue toward the poles above. A schematic of Thomson's circulation is shown in panel (b) of figure 2. Ferrel's circulation differed only in that the shallow indirect cell did not extend all the way to the poles. This is apparently because Ferrel was aware of observations which suggested that at very high latitudes the surface currents were toward the equator (Ferrel, 1856) while Thomson made no reference of this.

Thomson used the analogy of a stirred cup to argue that surface friction would draw fluid toward the poles, inducing a thermally indirect circulation at high latitudes (Thomson, 1857). Ferrel on the other hand, takes a much more mathematical approach. In his remarkable work of 1859 we see derivations of both the zonal and meridional components of the

Coriolis force as well as the geostrophic and thermal wind relations (Lorenz, 1967). We can understand his argument in modern terms as follows. Hadley's circulation required that the regions of surface easterlies must be balanced by a region of westerlies, so as not to rob the Earth of angular momentum. At the boundary between these two opposing surface currents geostrophic balance requires a maximum in pressure. Since frictional drag by the Earth acts to turn the geostrophic flow down the pressure gradient, the extra-tropical surface winds must acquire a poleward component (Lindzen, 1990). Friction due to wind shear can then allow for the required currents aloft. However, problems still remained with Ferrel's model. The indirect cell did not give the right transports of angular momentum and energy (Lorenz, 1967). Thus, while Ferrel's paper was a monumental advance in the field of meteorology the main problem with the Hadley model – the observation of a thermally indirect circulation at mid-latitudes – was yet to be explained.

2.3. Zonal asymmetries

Zonally asymmetric models of the mid-latitude circulation were proposed even before Ferrel and Thomson's theories attained prominence (Lorenz, 1967). Evidently however, a steady symmetric solution appealed much more to the meteorologists of the day, and these ideas were not widely adopted (Lorenz, 1967). In fact it was not until after the turn of the 20th century that the eddy driven mechanisms behind what we now call the Ferrel cell were deduced by Bjerknes and others. Again, it was new observations that forced the change in thinking. In the years 1896 - 1897 the United States Weather Bureau conducted a survey of high atmosphere clouds in order to try and deduce the winds aloft. The upper level thermally direct circulation required by the Ferrel/Thomson model was never found. Bigelow (1902) argued that this data invalidated Ferrel's 'canal' theory, and that it is a zonally asymmetric circulation that drives the meridional heat transport.

Jeffreys (1926) was the first to consider the transport of angular momentum by an asymmetric mid-latitude circulation. He showed that westerly winds at the surface must be maintained against friction by a positive flux of angular momentum, manifesting as a correlation between the eastward and northward velocities. Jeffreys argued that in a symmetric circulation, this momentum flux would be twenty times too small to account for the observed surface westerlies, and hence the general circulation must be asymmetric. The

final paradigm shift required before a three celled structure of the atmosphere could be accepted was the realisation that the mean circulation may not be a solution to the steady equations of motion. Since eddy motions transport significant amounts of momentum and heat, the balance requirements are not filled by the average circulation alone (Lorenz, 1967). This was seen by Bjerknes (1937) who believed that a zonally symmetric circulation, satisfying the balance requirements, could exist, but that it was unstable to zonal disturbances. The resultant unsteady circulation, he argued, could not be represented by any permanent system of flow (Bjerknes, 1937, p.328).

Thus, by the 1940s the mid-latitude circulation was beginning to be seen in terms of eddy concepts. The challenge for meteorologists now was to explain how eddies transport heat and momentum the way they do. It is this transport of momentum that is fundamental to determining how the surface winds are distributed on the globe. Thus we end the historical overview here, as more modern concepts will be discussed in detail in other sections. However, before we seek to understand the theoretical basis of the large-scale winds, it is necessary to get a picture of the structure of the general circulation through observations. With the advent of satellites and advanced data assimilation techniques, we now, like never before, can probe all regions of the atmosphere, to inform and verify our dynamical theories.

3. Observations of the general circulation

In contrast to the scant observations used to produce the theories of Hadley and Ferrel, modern meteorologists can readily access networks of ground based, radiosonde and, more recently, satellite observations in order to evaluate the state of the atmosphere (see Peixoto and Oort, 1984). While our evaluations of the large-scale atmospheric conditions are by no means perfect (see e.g. Andersson *et al.*, 2005), we will use analyses created with the observed data extensively in this section to get a feel for the mechanisms maintaining the general circulation – especially at mid-latitudes.

3.1. Zonally averaged circulation

Figure 3 shows the average distribution of zonal winds on the Earth. In the upper panel, contours of the zonally averaged zonal wind and temperature are displayed in the latitude-height plane. The most striking feature is the strong westerly jets that exist in the upper

troposphere around 30° north and south. To understand these wind maxima we use the concept of thermal wind. For an atmosphere close to geostrophic balance, the vertical wind shear can be related to horizontal pressure gradients via the thermal wind equation (e.g. Holton, 2004),

$$\frac{\partial u_g}{\partial p} = \frac{R}{f_0 p} \frac{\partial T}{\partial y} \qquad (3.1)$$

Here, T is the temperature, y is distance in the meridional direction, $R = 287 \text{ J kg}^{-1} \text{K}^{-1}$ is the dry gas constant, and we have used pressure, p, as the vertical co-ordinate. The geostrophic wind, u_g, is that based on an f-plane approximation. This is where the Coriolis parameter, defined by $f = 2\Omega \sin \varphi$, where Ω is the rotation rate of the Earth, and φ is the latitude, is assumed to have a constant value f_0. Equation (3.1) states that horizontal temperature gradients cause the wind to turn with height. Since the equator is warmer than the poles, the meridional temperature gradient is positive (negative) in the southern (northern) hemisphere. For both hemispheres this implies the geostrophic wind becomes more westerly with height. This can be seen in figure 3 – where the temperature contours (isotherms) are steepest is roughly the position of the upper level jet. In fact, calculation shows that in both the temporal mean, and at any given time, the large-scale flow above the boundary layer is in approximate thermal wind balance (Marshall and Plumb, 2007, ch. 7). Hence the upper level winds can be seen to be a consequence of the distribution of the winds at the surface, and the temperature distribution within the atmosphere. Our motivation for examining the causes of the surface winds is thus clear – they play a large role in determining the flow throughout the rest of the atmosphere. The temperature field, on the other hand, will be taken largely as given. The interaction of diabatic heating and dynamics is beyond the scope of this essay, and we will not consider it. Instead we simply note that the slopes of the isotherms in figure 3 are much steeper at mid-latitudes than in any other region. This is because the thermally direct circulations in the tropics and polar latitudes are much more efficient at transporting heat, and hence are able to homogenise the temperature field to a much greater degree (Marshall and Plumb, 2007, ch. 8).

Figure 3(b) shows the annual average of the zonal mean surface winds as a function of latitude. There are trade easterlies in the tropics, westerlies at mid-latitudes, and weak easterlies or no winds in polar regions. Consider the friction acting on these surface currents. Where there are easterlies, the Earth is rotating faster than the atmosphere above it. Thus the action of friction at these latitudes is to accelerate the atmosphere westwards, and

(a) Annual mean meridional cross-section of the atmosphere

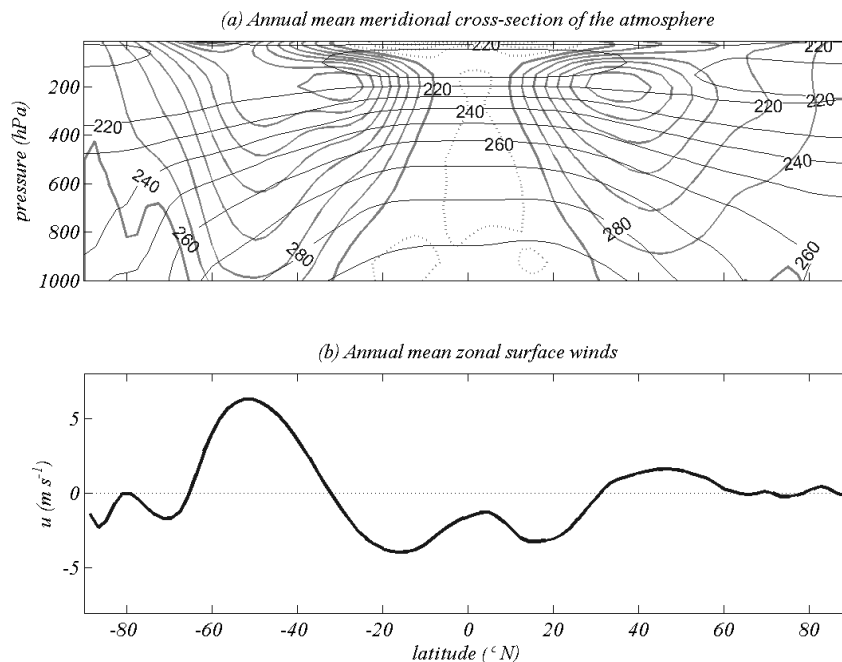*(b) Annual mean zonal surface winds*

Figure 3: Zonal and annual average picture of the zonal winds. Panel (a) shows zonal wind (unlabelled contours) and temperature (labelled contours) in the latitude-height plane. The contours for the winds are at intervals of 4 ms^{-1}, with the bold line marking the zero contours. Panel (b) shows the strength of the zonal surface winds as a function of latitude. Data is from the NCEP re-analysis (see Kalnay *et al.*, 1996).

slow down the Earth in its rotation. That is, there will be a flux of angular momentum from the Earth to the atmosphere in the tropics. Similarly, at mid-latitudes, the Earth must be a sink of angular momentum. Here, and throughout this essay, we understand the term 'angular momentum' to mean the component of angular momentum in the direction of the Earth's rotation. There are two components to this, planetary angular momentum, which depends on the Earth's rotation rate, and relative angular momentum, which depends on the velocity of the fluid. Now, when averaged over a year the amount of angular momentum in any latitude band is approximately constant. Thus, in order to balance the source and sink discussed above, there must be an atmospheric transport of angular momentum into the region of westerlies. Hence we see that if we can understand the observed angular momentum cycle of the atmosphere, we can explain the prevailing winds. For this reason angular momentum transport will feature heavily in the rest of this essay.

In the introduction we described the general circulation as made up of three overturning 'cells' in each hemisphere. Figure 4 shows the observational basis for this description. Plotted are contours of the 'Stokes mass stream-function', Ψ, for the meridional circulation. This function represents the southward mass flux from the top of the atmosphere to a given pressure, in units of 10^{10} kg s^{-1} (see Peixoto and Oort, 1984, for details). Thus contours of Ψ are parallel to the direction of average mass transport. The thermally direct Hadley circulation can be seen at low latitudes, with a very weak thermally direct cell also existing over the poles. At mid-latitudes there does exist, in the time and zonal mean at least, a thermally indirect cell, which we now call the Ferrel cell. This mean overturning extends all the way to the tropopause, and thus, as argued by Jeffreys (1926) cannot provide the correct horizontal transport of angular momentum on its own. Angular momentum must be

transported by the transient circulation − the eddies. In the next section we describe the method of Reynolds decomposition, used by Starr (1951) to analyse the transport properties of these eddies.

3.2. Momentum transport due to eddies

Transport of quantities due to eddies is fundamentally a problem in turbulence. However, no complete theory of turbulence exists, and thus we can only consider its effects using statistical approaches. In order to do this we use the method of Reynolds decomposition. Consider an arbitrary scalar variable α as being the sum of a mean and fluctuating part. We can take this mean either in time or in space. Thus we use the notation $\bar{\alpha}$ and $[\alpha]$ for the temporal and zonal means of our scalar, with corresponding fluctuations α' and α^*. It can be shown (Vallis, 2006) that the meridional transport of zonal momentum is proportional to the correlation $[uv]$. Furthermore, we can break this correlation into a mean and a fluctuating part, so that,

$$[uv] = [\bar{u}][\bar{v}] + [\overline{u^*}\,\overline{v^*}] + [\overline{u'v'}] \ (3.2)$$

These three terms correspond to the mean flow, stationary eddies, and transient eddies.

Figure 5 shows the observed transport of angular momentum in the atmosphere. The lower three panels show the contribution to the total transport by the transient eddies, stationary eddies and mean flow respectively. Clearly the transient eddies dominate in all regions above the boundary layer. Jeffreys' estimate made in 1926 is remarkably accurate − angular momentum transport by the mean flow is an order of magnitude smaller than that of the eddies. What is also evident however, is that poleward of 30° the mean flow actually transports angular momentum in the opposite direction to the total transport. Hence, as well as increasing the total transport tenfold, the transient eddies move the position of the upper level jets poleward. The contributions by
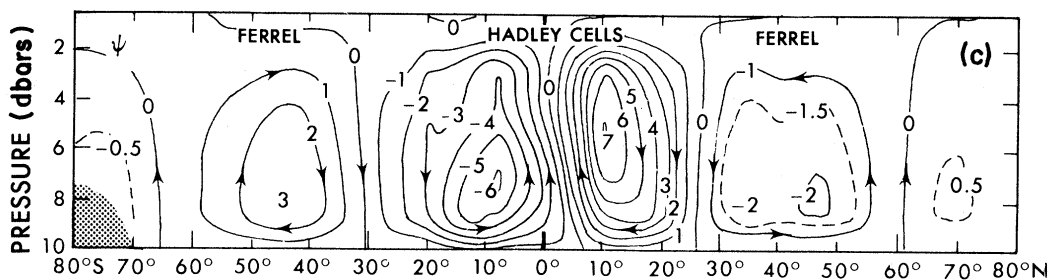

Figure 4: Stokes mass stream-function for the atmosphere in units of 10^{10} kg s-1. The vertical co-ordinate is pressure in decibels so that, $p_{dB} = 10 \log(p/p_s)$. Lines are streamlines of mean meridional mass transport. Source: Peixoto and Oort (1984).

stationary eddies shown in panel (c) are relatively small, except in the deep tropical stratosphere and some northern latitudes. The latter case arises because in these regions stationary Rossby waves can be excited by orography (i.e. the Himalayas) thus increasing the stationary eddy transport (Holton, 2004).

The top panel of figure 5 shows streamlines of the 'non-divergent' component of the atmospheric angular momentum transport (Peixoto and Oort, 1984). This corresponds to the total transport minus that of the mean meridional flow. In order to conserve mass, the mean meridional velocity cannot result in a net flux of angular momentum across latitude circles (Peixoto and Oort, 1984). Hence, its elimination allows us to see the transports that act to change the distribution of angular momentum in the atmosphere. It can be seen that in tropical latitudes the Earth is a source of angular momentum, which is then transported to the upper troposphere. The upper level dynamics takes this momentum poleward, to mid-latitudes where the flux of angular momentum is downwards, and the Earth is a sink. The dotted lines show contours of the average zonal velocity $[u]/\cos \varphi$. Note that angular momentum is transported from regions of low to regions of high relative velocity. That is, the transport is counter-gradient. Thus, if we consider turbulent effects as analogous to molecular viscosity, the 'turbulent viscosity' we require will be negative! This point was studied in detail by Starr (1968); however, more recently term negative viscosity has fallen out of favour (Held, 2000). Modern theories on the general circulation consider down gradient fluxes of potential vorticity as the driving factor behind the flow. We will seek to understand these dynamics in
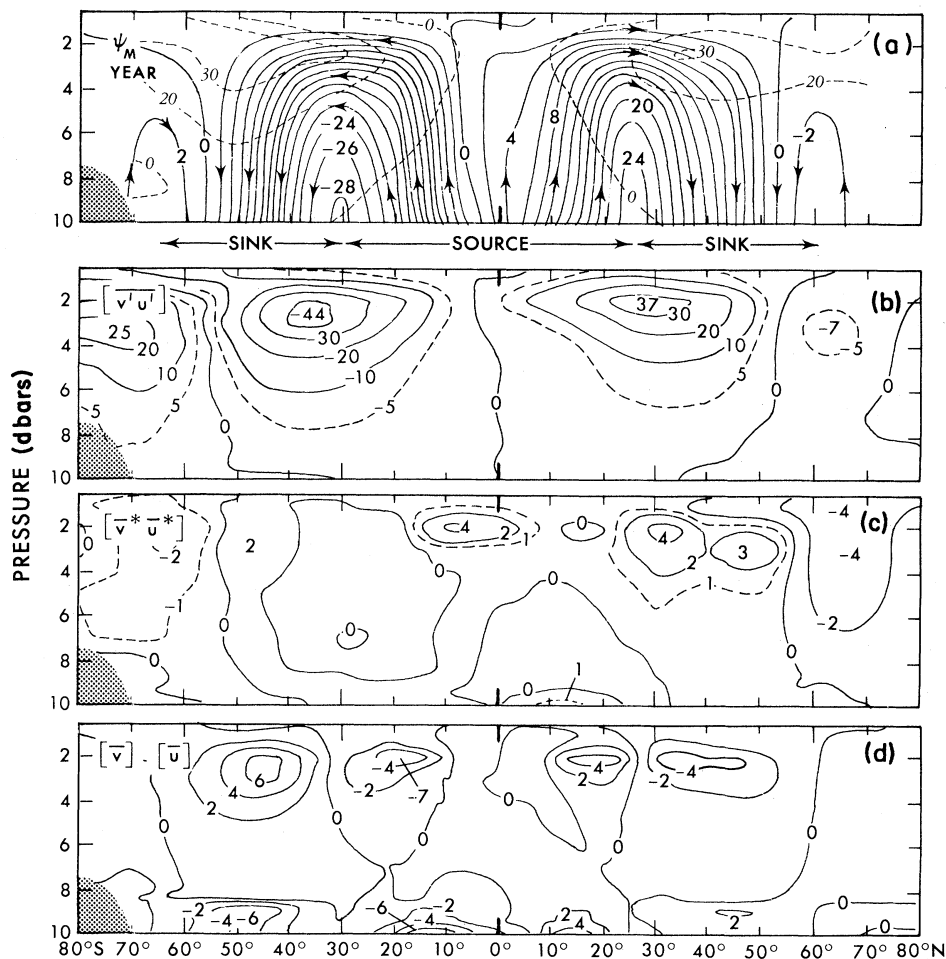

Figure 5: The meridional transport of angular momentum in the atmosphere. Panel (a) shows streamlines of the non-divergent component of angular momentum transport in units of 10^{18} kg m^{2}s^{-2}. The dotted lines are contours of the zonal velocity $[u] = \cos \varphi$, emphasising the counter gradient nature of the transport. Panels (b), (c) and (d) show horizontal momentum transport (m^{2}s^{-2}) by the mean circulation, the stationary eddies, and the transient circulation respectively. The vertical axis is pressure in decibels, as in figure 4. Source: Peixoto and Oort (1984).

section 4.2. But first we consider some simpler models in order to see how these counter-gradient momentum fluxes are maintained.

4. Conceptual models of the extra-tropical circulation

As we have seen in the last section, the transport of angular momentum appears to be of prime importance to the zonal surface currents. As was originally argued by Jeffreys (1926), the angular momentum at a given latitude can be changed in three ways – frictional exchange with the Earth, pressure differences between the East and West side of orography ('mountain torques'), and the flux of momentum from other latitudes. Now, mountain torques are generally smaller in magnitude, and act in the same direction as friction (Peixoto and Oort, 1984). Thus, in order for a barotropic jet to be maintained in statistical steady state there must be a meridional flux of momentum to overcome frictional loss at the surface. Where does this flux come from? In the next section we show how large scale eddies on a rotating planet act to create the required flux of momentum. For the time being, we limit our discussion to a barotropic atmosphere, in which the eddies are the result of some externally forced 'stirring'. The barotropic case is instructive to consider before we understand the full problem, in which baroclinic instability is the cause of the stirring.

4.1. Momentum transport by Rossby waves

The largest scale disturbances in mid-latitudes are planetary Rossby waves. These waves are due to the change with latitude of the local rotation rate. The simplest way to see how they arise is to consider a homogeneous, inviscid fluid of constant depth. Under these assumptions it can be shown that a parcel of fluid conserves its absolute vorticity as it moves (Holton, 2004),

$$\frac{D(\zeta + f)}{Dt} = 0 \qquad (4.1)$$

Here f is the Coriolis parameter, and $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the relative vorticity. If a fluid parcel is pushed from its rest position toward the north, its latitude will increase, and thus the value of f will also increase. Hence, there must be a compensating decrease in ζ, that is, a clockwise turning of the flow. Similarly, for southward displacements an anticlockwise rotation is induced. The net result is that a Coriolis parameter that increases to the North supports eastward propagating waves.

Now, since the fluid is horizontally non-divergent, we can define a stream-function such that, $(u,v) = \left(-\frac{\partial \Psi}{\partial y}, \frac{\partial \Psi}{\partial x}\right)$. Thus (4.1) becomes,

$$\frac{D}{Dt}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] + \beta v = \frac{D}{Dt}\nabla^2\Psi + \beta\frac{\partial \Psi}{\partial x} = 0$$

where we have used the β-plane approximation, $f = f_0 + \beta y$. Assuming the solution is a perturbation about a mean state that is at rest, we can linearise the above equation so that,

$$\frac{\partial \nabla^2\Psi}{\partial t} + \beta\frac{\partial \Psi}{\partial x} = 0 \qquad (4.2)$$

Now, since (4.2) is a constant co-efficient partial differential equation we need only consider solutions of the form,

$$\Psi = \Re\left\{\Phi e^{i(kx + ly - \omega t)}\right\} \qquad (4.3)$$

For a single normal mode component, the transport of angular momentum by this wave will be proportional to,

$$\overline{[uv]} = -\overline{[\Psi_x \Psi_y]} = -\frac{kl}{2}\Phi^2$$

Thus we see that the meridional transport of angular momentum is governed by the product kl. The sign of this quantity determines the direction of the meridional eddy flux of momentum. Substituting our plane wave solution into the equation, we derive the dispersion relation for the simplest type of Rossby waves,

$$\omega = -\frac{\beta k}{k^2 + l^2} \qquad (4.4)$$

Taking the derivative with respect to l we find the meridional group velocity, G_y,

$$G_y = \frac{\partial \omega}{\partial l} = -\frac{2\beta kl}{\left(k^2 + l^2\right)^2} \qquad (4.5)$$

Hence, the direction of eddy flux of momentum depends on the sign of the group velocity of the waves. Now, the group velocity of a wave can be shown to be the rate at which it transports energy (Holton, 2004), and thus must be directed away from the source. North of the disturbance, G_y must be positive and $\overline{uv}$ negative – momentum is transported toward the source region. Similarly, the momentum flux from the south is also toward the disturbance. Hence there is a convergence of westerly momentum in the region of the disturbance. Physically we see this means that in order for the Rossby waves to transport

energy out of the source region, they must converge momentum into it (Thompson, 1971).

It is interesting to note how the momentum flux manifests physically. Consider lines of constant phase at some given time,

$$kx + ly = \phi$$

$$\Rightarrow y = \frac{k}{l}x - \frac{\phi}{l}.$$

Since kl is positive (negative) on the north (south) side of the disturbance, this gives rise to a tilting of the troughs and ridges of mid-latitude Rossby waves. Figure 6 shows the resultant 'bow-shaped' eddies, that work to converge momentum into the region of stirring. Note that if the velocities were in the opposite direction (clockwise rotation) the momentum fluxes would still be the same. Thus both high and low pressure systems can contribute to the sharpening of the jet.

4.2. Baroclinic instability

In the previous section we have shown the importance of horizontal angular momentum transports to the maintenance of the mid-latitude westerlies, and indeed the general circulation. We have also demonstrated how this transport manifests as a barotropic response to turbulent mixing in the extra-tropics. The ultimate cause of this large-scale eddy motion is baroclinic instability. This process, sometimes termed 'sloping convection' is the release of potential energy associated with warm and cold fluids lying beside one another (Houghton, 2002). This
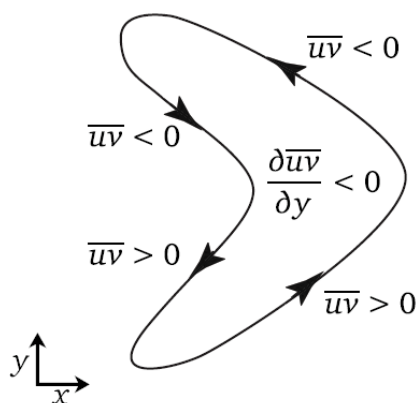
Figure 6: Tilting of the troughs of a mid-latitude eddy as predicted by a consideration of the momentum flux of Rossby waves. The contribution of each quadrant of the velocity correlation is a momentum flux toward the centre of the eddy. Thus, eddies sharpen, rather than diffuse an eastward jet. Source: Vallis (2006).

type of instability was first studied, in an idealised way by Eady (1949) and Charney (1947) who analysed the behaviour of a baroclinic fluid under the quasi-geostrophic approximation. A detailed treatment of baroclinic instability is beyond the scope of this essay, however, following Marshall and Plumb (2007) we will discuss the concept from the point of view of potential energy.

Consider the situation in figure 7, where a light fluid overlies a denser one. If the interface is sloping, one would expect the fluid to adjust by overturning in the plane of the page, thus destroying the interface slope. However, in the case of a rapidly rotating fluid, geostrophic adjustment prevents the fluid flowing at right angles to the geopotential height contours (Holton, 2004). Thus the fluid is constrained by rotation, and cannot destroy the interface slope directly. Instead, instability develops via fluid being exchanged along an incline, with an angle smaller than that of the interface, in the so-called 'wedge of instability' (Pedlosky, 1987).

We can see this more explicitly by thinking about the potential energy of the fluid. The gravitational potential energy of the fluid can be written,

$$\text{GPE} = \int \rho z \, dV \qquad (4.6)$$

where ρ is the density, z is the vertical height, and the integral is taken over the entire volume of the fluid. Now consider how this changes as we rearrange fluid parcels. If we take a parcel from the 'wedge' within the light fluid and exchange it for one within the 'wedge' of heavy fluid we see that we have moved the lighter fluid up, and the denser fluid down. The centre of mass of the fluid has been lowered, and the integral in (4.6) has decreased. This decrease in GPE must be associated with an increase in kinetic energy. This is the energy source for the mid-latitude eddies (Marshall and Plumb, 2007). For any interchange of fluid parcels other than the one just described, the potential energy increases. Hence the only method available to release energy associated with sloping density surfaces within a rapidly rotating fluid is 'sloping convection', where there is mixing of the fluid within the 'wedge of instability' (Marshall and Plumb, 2007). This process explains the stirring we were required to invoke in our barotropic discussion of mid-latitude dynamics.

4.3. Current work and open problems

A large amount of work relating to the atmospheric general circulation in the latter part of last century was done through quasi-

geostrophic scaling (Hoskins, 1991), the same approximations used in the original treatment of baroclinic instability laid down by Eady and Charney. There are, however, disadvantages to this, as the assumptions required can be restrictive (Schneider, 2006). One of the key omissions from this theory is that the temperature field must be prescribed. Geostrophic eddies in the extra-tropics can be seen as a result of the strong temperature gradients there. However the strong temperature gradients themselves are partly a result of the dynamics, and thus our separation into cause and effect is somewhat artificial.

Recently, a view has emerged that the best way to analyse the large-scale dynamics of the atmosphere is by looking at Ertel potential vorticity (PV) from the perspective of isentropic co-ordinates (Koh and Plumb, 2004). The reasons for this are that (1), PV is materially conserved and (2), it cannot be transported across isentropic surfaces (Haynes and McIntyre, 1990). Thus, in isentropic coordinates, the PV in any horizontal layer of the atmosphere is constant unless that layer intersects the surface – the surface is the only PV source or sink. Ertel potential vorticity depends on temperature, and thus the resultant equations include the thermodynamic variables. Hence, in principle, a complete theory of the (dry) general circulation, including its interaction with the temperature field could be achieved with a theory of the transport of potential vorticity (Schneider, 2006). However, this must involve a description of the turbulent flux of PV, which is currently the subject of active research. One might ask whether we have gained anything by taking the PV view. After all, flux-gradient type closures for the mid-latitude eddies were

being proposed as early as the 1930s (Lorenz, 1967). However, since PV is a conserved quantity, we might have some more confidence that the flux is down-gradient, and that we can find a closure that can be theoretically justified (Wardle and Marshall, 2000). Nevertheless, it is clear that there are still some unanswered questions in our understanding of the general circulation, and the history outlined at the beginning of this essay is not complete.

5. Summary

Problems relating to the general circulation have been central to meteorological research since its beginnings, and, as we have seen, there is still work being done in this field today. The modern picture is that of a three-celled circulation. There is a thermally direct cell in the tropics and at polar latitudes, while the mean circulation at mid-latitudes is thermally indirect. Popular acceptance of this description did not arrive until after the turn of the 20th century (Bigelow, 1902; Bjerknes, 1937). Previous to this, the prevailing view was that of a two-celled circulation (Thomson, 1857; Ferrel, 1859), which itself superseded the original one cell description (Hadley, 1735).

The reason for this difficulty in characterising the general circulation has much to do with the unsteady nature of the mid-latitude flow. The 'Ferrel' cell is driven not by the mean winds, but by eddies, which affect much of the transports of heat and momentum through the extra-tropics. Thus a theory of the general circulation cannot be successful unless it takes into account these zonal and temporal asymmetries.

Similarly, we have seen that the mid-latitude surface currents are closely coupled to the transports of angular momentum and thus are also driven by the large scale eddies. The interaction between these eddies and the mean flow is complex, however, much can be learned from a linear, barotropic perspective. When a rotating atmosphere is stirred, waves due to the gradient of planetary vorticity are excited. If these Rossby waves propagate outside the region of the disturbance before they break they will result in momentum converging toward the stirring. Thus the mid-latitude westerlies can be maintained against friction by the action of eddies, coupled with a gradient in planetary vorticity.

To see where the eddies come from we must also look at the temperature distribution. If meridional temperature gradients at the surface are sufficiently strong, quasi-geostrophic theory tells us the atmosphere can become
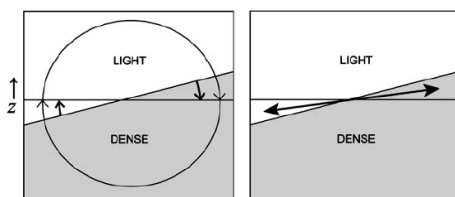
Figure 7: A two-layer fluid in the case of no-rotation (left) and when rotational effects are strong (right). On the left the potential energy of the sloping interface is released directly by the fluid overturning in the plane of the page. In the rapidly rotating case, however, rotation constrains the motion, and the energy can only be released by exchange of fluid in the 'wedge of instability'. Source: Marshall and Plumb (2007).

baroclinically unstable. These instabilities release available potential energy associated with the temperature field into eddy kinetic energy. Since the meridional temperature gradients are strongest at mid-latitudes this is the region of most vigorous eddy activity.

Recent work on the mid-latitude circulation has generalised quasi-geostrophic theory using the so-called `PV-θ view' (Hoskins, 1991). In this framework potential vorticity is analysed in isentropic co-ordinates, allowing many of the main results in quasi-geostrophy to be derived without the more restrictive assumptions. Thus if this potential vorticity flux can be understood, a complete understanding of the mid-latitude circulation can be developed. However, theories of the eddy transport of potential vorticity are as yet not satisfactory. Thus, work is still to be done to understand the deep theory behind the seemingly simple question of why the extra-tropics are dominated by westerly surface winds.

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Reconstructing Historical Rainfall Averages for the Murray-Darling Basin

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1.   Introduction

The Murray-Darling Basin (MDB) is, at the time of writing, in a severe drought. In its Murray River System Drought Update for April 2009 (MDBA, 2009), the Murray-Darling Basin Authority wrote that "The outlook for the 2009-10 water year is not good, with record low inflows for the past 3 months, low storage levels and a rainfall outlook that indicates drier than average conditions for the next 3 months over the southern Murray-Darling Basin."

The National Climate Centre (NCC) has been producing national gridded analyses of monthly rainfall (Jones and Weymouth, 1997) since the mid-1990s, and keeping track of rainfall deficiencies across the MDB via gridded rainfall percentile calculations since at least the late 1990s. These analyses have a resolution of 0.25° in latitude and longitude.
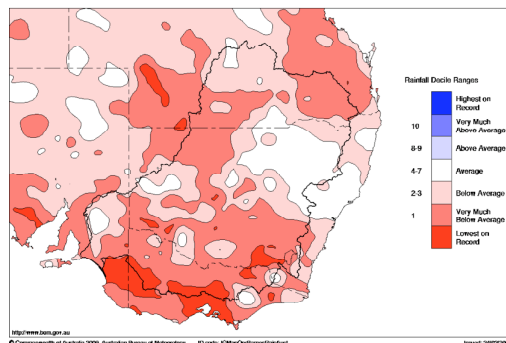

Figure 1: Rainfall deciles for the seven years 2002-2008, based on monthly gridded rainfall analyses (Jones and Weymouth, 1997) from 1900 to 2008. The MDB boundary is shown as a solid black line.

Figure 1 shows rainfall deciles across the MDB for the seven-year period 2002-2008, based on these monthly analyses. Across most of the southern half of the MDB the seven-year totals have been "very much below average" (i.e., below the 10th percentile of the historical distribution), and in no part of the MDB has the seven-year total been "above average" (i.e., above the 70th percentile).

These gridded monthly analyses are also used to produce area-averaged values. These area-averaged values are calculated by averaging the grid-point values, with a weighting factor based on latitude to take account of meridional convergence. They largely supersede the prior calculations based on Bureau of Meteorology rainfall districts (Jones and Beard, 1998). Figure 2 shows annual rainfall totals[1] averaged across the MDB, based on the NCC monthly analyses. The driest year in the time series is 1902 (256 mm), while the wettest year is 1956 (821 mm). The average is 476 mm.

In terms of seven-year summations, the 2002-2008 period ranks sixth worst in area-averaged terms (see Table 1). From the table, it is clear that there is essentially only one previous period (the mid 1930s to the mid 1940s) in the period covered by the NCC monthly analyses (1900-present) comparable with the current dry period in the MDB.

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[1] These data are accessible via http://www.bom.gov.au/ cgi-bin/climate/change/timeseries.cgi.